Journal of Thermophysics and Heat Transfer
Vol. 21, No. 2, April–June 2007

Optimal Design of Tube Banks in Crossflow Using Entropy Generation Minimization Method

W. A. Khan*
COMSTATS (Commission on Science and Technology for Sustainable Development in the South) Institute of Information Technology, Abbottabad, Pakistan
and
J. R. Culham† and M. M. Yovanovich‡
University of Waterloo, Waterloo N2L 3G1, Canada

DOI: 10.2514/1.26824

An entropy generation minimization method is applied as a unique measure to study the thermodynamic losses caused by heat transfer and pressure drop for a fluid in cross flow with tube banks. The use of entropy generation minimization allows the combined effect of heat transfer and pressure drop to be assessed through simultaneous interaction with the tube bank. A general dimensionless expression for the entropy generation rate is obtained by considering a control volume around a tube bank and applying conservation equations for mass and energy with entropy balance. Analytical/empirical correlations for heat transfer coefficients and friction factors are used, where the characteristic length is used as the diameter of the tubes and reference velocity used in Reynolds number and pressure drop is based on the minimum free area available for the fluid flow. Both inline and staggered arrangements are studied and their relative performance is compared for the same thermal and hydraulic conditions. A parametric study is also performed to show the effects of different design variables on the overall performance of tube banks. It is shown that all relevant design parameters for tube banks, including geometric parameters and flow conditions, can be simultaneously optimized.

Nomenclature

A	=	surface area of a single tube, m^2
A_t	=	total heat transfer area, m^2
C_1	=	constant defined in Eqs. (15)
D	=	tube diameter, m
e	=	specific energy, W/kg
f	=	friction factor
g, l	=	equality and inequality constraints
h_{avg}	=	average heat transfer coefficient of tubes, $\mathrm{W/m^2 \cdot K}$
j	=	number of imposed constraints
K_1	=	constant defined in Eqs. (17)
k	=	thermal conductivity, $\mathrm{W/m \cdot K}$
L	=	length of tube, m
$\mathcal{L}$	=	Lagrangian function
$\dot{m}$	=	mass flow rate, kg/s
N	=	total number of tubes $\equiv N_T N_L$
N_L	=	number of rows in streamwise direction
N_s	=	dimensionless entropy generation rate $\equiv \dot{S}_{\mathrm{gen}}/(Q^2 U_{\max}/k_f \nu T_a^2)$
N_T	=	number of rows in spanwise direction
Nu_D	=	Nusselt number based on tube diameter
n	=	number of design variables
P	=	pressure, Pa
Pr	=	Prandtl number $\equiv \nu/\alpha$
Q	=	heat transfer rate over the boundaries of CV, W
Re_D	=	Reynolds number based on maximum velocity in minimum flow area $\equiv DU_{\max}/\nu$
$\dot{S}_{\mathrm{gen}}$	=	total entropy generation rate, W/K
S_D	=	diagonal pitch, m
$\mathcal{S}_D$	=	dimensionless diagonal pitch $\equiv S_D/D$
S_L	=	tube spacing in streamwise direction, m
$\mathcal{S}_L$	=	dimensionless streamwise pitch $\equiv S_L/D$
S_T	=	tube spacing in spanwise direction, m
$\mathcal{S}_T$	=	dimensionless spanwise pitch $\equiv S_T/D$
T	=	absolute temperature, K
U_{app}	=	approach velocity, m/s
$U_{\max}$	=	maximum velocity in minimum flow area, m/s
v	=	specific volume of fluid, m^3/kg
$\dot{W}_{\mathrm{cv}}$	=	energy transfer by work across the boundaries of CV, J
x_i	=	design variables
γ	=	aspect ratio $\equiv L/D$
ν	=	kinematic viscosity of fluid m^2/s
ρ	=	fluid density kg/m^3

Subscripts

a	=	ambient
f	=	fluid
in	=	inlet of CV
out	=	exit of CV
T	=	thermal
w	=	wall

Presented as Paper 599 at the 44th AIAA Aerospaces Meeting and Exhibit, Reno, Nevada, 9–12 January 2006; received 27 July 2006; revision received 3 October 2006; accepted for publication 3 October 2006.

*Associate Professor, Department of Mathematics, Post Code 22060. Member AIAA.

†Associate Professor, Director, Microelectronics Heat Transfer Lab., Department of Mechanical Engineering.

‡Distinguished Professor Emeritus, Department of Mechanical Engineering. Fellow AIAA.

I. Introduction

BECAUSE of extensive use of high performance compact heat exchangers, an optimal design of tube banks is very important. Compact heat exchangers are found in numerous applications, such as an automobile radiator, an oil cooler, a preheater, an air-cooled steam condenser, a shell and tube type heat exchanger, and the evaporator of an air conditioning system. Tube banks are usually arranged in an inline or staggered manner, where one fluid moves across the tubes, and the other fluid at a different temperature passes through the tubes. In this study, the authors are specifically interested in determining an optimal design of the tube banks in cross flow using an entropy generation minimization method. The crossflow correlations for the heat transfer and pressure drop are employed to calculate entropy generation rate.

A. Literature Review

A careful review of existing literature reveals that most studies are related to the optimization of plate heat exchangers and only few studies are related to tube heat exchangers.

To the authors' knowledge, McClintock [1] was the first one who employed the concept of irreversibility for estimating and minimizing the usable energy wasted in heat exchangers design. Bejan [2–5] presented an optimum design method for balanced and imbalanced counterflow heat exchangers. He proposed the use of a "number of entropy production units" N_s as a basic parameter in describing heat exchanger performance. This method was applied to a shell and tube regenerative heat exchanger to obtain the minimum heat transfer area when the amount of units was fixed. Later on, Aceves-Saborio et al. [6] extended that approach to include a term to account for the energy of the heat exchanger material. Grazzini and Gori [7], Sekulic [8], Zhang et al. [9], Ordonez and Bejan [10], and Bejan [11,12] demonstrated that the optimal geometry of a counterflow heat exchanger can be determined based on a thermodynamic optimization subject to volume constraint. Yilmaz et al. [13] first recalled and discussed the need for the systematic design of heat exchangers using a second-law-based procedure and then presented second-law-based performance evaluation criteria for heat exchangers. Entropy generation rate is generally used in a dimensionless form. Unuvarn and Kargici [14] and Peters and Timmerhaus [15] presented an approach for the optimum design of heat exchangers. They used the method of steepest descent for the minimization of annual total cost. They observed that this approach is more efficient and effective to solve the design problem of heat exchangers. Optimization of plate-fin and tube-fin crossflow heat exchangers was presented by Shah et al. [16,17] and Van den Bulck [18]. They employed optimal distribution of the UA value across the volume of crossflow heat exchangers and optimized different design variables like fin thickness, fin height, and fin pitch.

Cylinder geometry was optimized in a paper by Poulikakos and Bejan [19] and Khan et al. [20]. After the general formula was derived using the entropy generation minimization (EGM) method analytical methods and graphical results were developed that resulted in the optimum selection of the dimensions of several different fin configurations. Bejan et al. [21] showed that EGM may be used by itself in the preliminary stages of design, to identify trends and the existence of optimization opportunities. In two different studies, Stanescu et al. [22], and Matos et al. [23] demonstrated that the geometric arrangement of tubes/cylinders in crossflow forced convection can be optimized for maximum heat transfer subject to overall volume constraint. They used FEM to show the optimal spacings between rows of tubes. Vargas et al. [24] documented the process of determining the internal geometric configuration of a tube bank by optimizing the global performance of the installation that uses the crossflow heat exchanger.

B. Assumptions

The following assumptions are used in this study:

1) The tube bank is insulated from its surroundings.
2) The surface of tubes is plain.
3) The flow is 2-D, steady, and laminar.
4) The fluid is Newtonian and incompressible with constant properties.
5) The conduction along tube walls is negligible.
6) The radiation heat transfer from tubes is negligible.
7) The potential and kinetic energy changes are negligible.

II. Model Development

Consider the inline tube bank in crossflow delineated by a control volume (CV) in Fig. 1. The sides of this control volume can be regarded as impermeable, adiabatic, and shear free. Fluid I moves across the tubes while fluid II at a different temperature passes through the tubes.

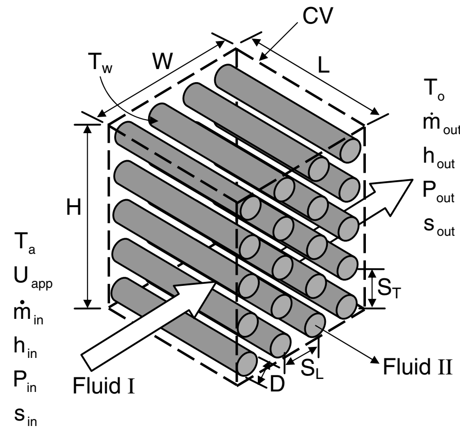

Fig. 1 Control volume for calculating $\dot{S}_{\mathrm{gen}}$ for tube banks.

In this study, the authors are concerned only with fluid I. The approach velocity of the fluid I is U_{app} and the ambient temperature is T_a. The temperature of the tube wall is T_w. The properties of the fluid are represented by e_{in}, P_{in}, s_{in} at the inlet and by e_{out}, P_{out}, s_{out} at the outlet, respectively. The irreversibility of this system is also due to heat transfer across the nonzero temperature difference $T_w - T_a$ and due to the total pressure drop across the tube bank. The mass rate balance for the CV, shown in Fig. 1, gives

$$\frac{dm_{\mathrm{cv}}}{dt} = \dot{m}_{\mathrm{in}} - \dot{m}_{\mathrm{out}} \tag{1}$$

For steady state, it reduces to

$$\dot{m}_{\mathrm{in}} = \dot{m}_{\mathrm{out}} = \dot{m} \tag{2}$$

The first law of thermodynamics for the same CV can be written as

$$\frac{dE_{\mathrm{cv}}}{dt} = \mathcal{Q} - \dot{W}_{\mathrm{cv}} + \dot{m}_{\mathrm{in}}(e_{\mathrm{in}} + P_{\mathrm{in}}v_{\mathrm{in}}) - \dot{m}_{\mathrm{out}}(e_{\mathrm{out}} + P_{\mathrm{out}}v_{\mathrm{out}}) \tag{3}$$

where dE_{cv}/dt is the time rate of change of energy within the CV and for steady state, $(dE_{\mathrm{cv}}/dt) = 0$. The specific energy e is the sum of specific internal, kinetic, and potential energies. Because of continuity and same elevation of the CV, $V_{\mathrm{in}} = V_{\mathrm{out}}$ and $z_{\mathrm{in}} = z_{\mathrm{out}}$, so that the kinetic and potential energy terms will drop out. Therefore, $e_{\mathrm{in}} = u_{\mathrm{in}}$ and $e_{\mathrm{out}} = u_{\mathrm{out}}$. The only work is flow work at the inlet and exit of the CV, and so the term $\dot{W}_{\mathrm{cv}}$ also drops out. Thus the energy rate balance reduces to

$$\mathcal{Q} = \dot{m}[\underbrace{(u_{\mathrm{out}} + P_{\mathrm{out}}v_{\mathrm{out}})}_{h_{\mathrm{out}}} - \underbrace{(u_{\mathrm{in}} + P_{\mathrm{in}}v_{\mathrm{in}})}_{h_{\mathrm{in}}}] \tag{4}$$

The combination of specific internal and flow energies is defined as specific enthalpy; therefore, the energy rate balance reduces further to

$$\mathcal{Q} = \dot{m}(h_{\mathrm{out}} - h_{\mathrm{in}}) \tag{5}$$

From the second law of thermodynamics

$$\frac{dS_{\mathrm{cv}}}{dt} = \dot{m}(s_{\mathrm{in}} - s_{\mathrm{out}}) + \frac{\mathcal{Q}}{T_w} + \dot{S}_{\mathrm{gen}} \tag{6}$$

For steady state, $dS_{\mathrm{cv}}/dt = 0$, and so the entropy rate balance reduces to

$$\dot{S}_{\mathrm{gen}} = \dot{m}(s_{\mathrm{out}} - s_{\mathrm{in}}) - \frac{\mathcal{Q}}{T_w} \tag{7}$$

Gibbs equation $[dh = Tds + (1/\rho)dP]$ can be written as

$$h_{\rm out} - h_{\rm in} = T_a(s_{\rm out} - s_{\rm in}) + \frac{1}{\rho}(P_{\rm out} - P_{\rm in}) \tag{8}$$

Combining Eqs. (5) and (8), we get

$$\mathcal{Q} = \dot{m}T_a(s_{\rm out} - s_{\rm in}) - \frac{\dot{m}\Delta P}{\rho} \tag{9}$$

Combining Eqs. (7) and (9), we get

$$\dot{S}_{\rm gen} = \left(\frac{\mathcal{Q}^2}{T_a T_w}\right)R_{\rm tube} + \frac{\dot{m}\Delta P}{\rho T_a} \tag{10}$$

where $R_{\rm tube}$ is the tube wall thermal resistance, $\dot{m}$ is the mass flow rate through the tubes, and ΔP is the pressure drop across the tube bank and can be written as

$$R_{\rm tube} = \frac{\Delta T}{\mathcal{Q}} = \frac{1}{h_{\rm avg}A} \tag{11}$$

$$\dot{m} = \rho U_{\rm app} N_T S_T L \tag{12}$$

$$\Delta P = N_L f\left(\frac{1}{2}\rho U_{\rm max}^2\right) \tag{13}$$

Khan [25] has developed the following analytical correlation for dimensionless heat transfer coefficient for the tube banks:

$$Nu_D = C_1\left(C_2 Re_D^{1/2} Pr^{1/3} + 0.001 Re_D\right) \tag{14}$$

where Re_D is the Reynolds number defined as $Re_D = DU_{\rm max}/\nu$ and C_1 and C_2 are the constants that depend upon the longitudinal and transverse pitch ratios, arrangement of the tubes, and thermal boundary conditions. For isothermal boundary condition, they are given by

$$C_1 = \begin{cases} \frac{1.23+1.47N_L^{1.25}}{1.72+N_L^{1.25}} & \text{inline} \\ \frac{1.21+1.64N_L^{1.44}}{1.87+N_L^{1.44}} & \text{staggered} \end{cases} \tag{15}$$

$$C_2 = \begin{cases} \frac{-0.016+0.6\mathcal{S}_L^2}{0.4+\mathcal{S}_L^2} & \text{inline} \\ (0.588+0.004\mathcal{S}_T)\left(0.858+0.04\mathcal{S}_T-0.008\mathcal{S}_T^2\right)^{1/\mathcal{S}_L} & \text{staggered} \end{cases} \tag{16}$$

Equations (15) and (16) are valid for $1.25 \le \mathcal{S}_L \le 3$ and $1.05 \le \mathcal{S}_T \le 3$.

Žukauskas and Ulinskas [26] collected data, from a variety of sources, about friction factors for flow in the inline and staggered arrangements having many rows and plotted them in the form Eu/K_1 versus Re_D, where K_1 is a correction factor accounting for geometry. They fitted these plots by inverse power series relationships and recommended more than 30 correlations for friction and correction factors depending on the transverse and longitudinal pitch ratios, the Reynolds number range and the type of arrangement. Khan [27] digitized their experimental data and fitted into single correlations for the friction and correction factors for each arrangement. These correlations can be used for any pitch ratio $1.25 \le \mathcal{S}_L$ or $\mathcal{S}_T \le 3.0$ and Reynolds number in the laminar flow range. They are

$$f = \begin{cases} K_1[0.233 + 45.78/(\mathcal{S}_T - 1)^{1.1} Re_D] & \text{inline} \\ K_1\left[378.6/\mathcal{S}_T^{13.1/\mathcal{S}_T}\right]/Re_D^{0.68/\mathcal{S}_T^{1.29}} & \text{staggered} \end{cases} \tag{17}$$

where K_1 is a correction factor depending upon the flow geometry and arrangement of the pins. It is given by

$$K_1 = \begin{cases} 1.009\left(\frac{\mathcal{S}_T-1}{\mathcal{S}_L-1}\right)^{1.09/Re_D^{0.0553}} & \text{inline} \\ 1.175\left(\mathcal{S}_L/\mathcal{S}_T Re_D^{0.3124}\right) + 0.5Re_D^{0.0807} & \text{staggered} \end{cases} \tag{18}$$

The velocity $U_{\rm max}$, used in Eq. (13) and in the definition of Reynolds number, represents the maximum average velocity seen by the bank as flow accelerates between tubes, and is given by

$$U_{\rm max} = \max\left\{\frac{\mathcal{S}_T}{\mathcal{S}_T - 1}U_{\rm app}, \frac{\mathcal{S}_T}{\mathcal{S}_D - 1}U_{\rm app}\right\} \tag{19}$$

where $\mathcal{S}_D = \sqrt{\mathcal{S}_L^2 + (\mathcal{S}_T/2)^2}$ is the dimensionless diagonal pitch.

Using Eqs. (11–14), the entropy generation rate can be simplified to

$$\dot{S}_{\rm gen} = \frac{\mathcal{Q}^2/T_a T_w}{C_1 N\pi L k_f Re_D^{1/2} Pr^{1/3}} + \frac{Nf\rho U_{\rm max}^3(\mathcal{S}_T - 1)L}{2T_a} \tag{20}$$

The first term on the RHS represents the entropy generation rate due to heat transfer, whereas the second term represents the entropy generation rate due to fluid flow. For external flow, Bejan [21] used the term $\mathcal{Q}^2 U_{\rm max}/k_f \nu T_a^2$ to nondimensionalize the entropy generation rate in Eq. (19). And so the dimensionless entropy generation rate can be written as

$$N_s = \frac{T_a/T_w}{C_1 N\pi\gamma Re_D^{3/2} Pr^{1/3}} + \frac{1}{2}fN\gamma B Re_D^2(\mathcal{S}_T - 1) \tag{21}$$

where $B = \rho\nu^3 k_f T_a/\mathcal{Q}^2$ is a fixed dimensionless duty parameter that accounts for the importance of fluid friction irreversibility relative to heat transfer irreversibility. Equation (20) shows that, for the given volume of the tube bank and heat duty, the dimensionless entropy generation rate depends on ambient and wall temperatures, total number of tubes, longitudinal and transverse pitch ratios, Reynolds and Prandtl numbers, and aspect ratio. After fixing ambient and wall temperatures, all these parameters depend on tube diameter and the approach velocity for given longitudinal and transverse pitches.

III. Optimization Procedure

The problem considered in this study is to minimize the dimensionless entropy generation rate, given by Eq. (20), for the optimal overall performance of the tube bank. If $f(\boldsymbol{x})$ represents the dimensionless entropy generation rate that is to be minimized subject to equality constraints $g_j(x_1, x_2, \ldots, x_n) = 0$ and inequality constraints $l_k(x_1, x_2, \ldots, x_n) \ge 0$, then the complete mathematical formulation of the optimization problem may be written in the following form:

$$\text{minimize } f(\boldsymbol{x}) = N_s(\boldsymbol{x}) \tag{22}$$

subject to the equality constraints

$$g_j(\boldsymbol{x}) = 0, j = 1, 2, \ldots, m \tag{23}$$

and inequality constraints

$$l_j(\boldsymbol{x}) \ge 0, j = m+1, \ldots, n \tag{24}$$

where g_j and l_j are the imposed equality and inequality constraints and $\boldsymbol{x}$ denotes the vector of the design variables $(x_1, x_2, x_3, \ldots, x_n)^T$. In this study, the design variables $\boldsymbol{x}$ are

$$\boldsymbol{x} = [D, H, W, L, U_{\rm app}, Q]$$

Inequality constraints are

$$D(mm) \ge 10 \tag{25}$$

$$1.25 \le \mathcal{S}_L \le 3 \tag{26}$$

$$1.25 \le \mathcal{S}_T \le 3 \tag{27}$$

$$\gamma \ge 20 \tag{28}$$

The objective function can be redefined by using Lagrangian function as follows:

$$\mathcal{L}(\boldsymbol{x}, \lambda, \chi) = f(\boldsymbol{x}) + \sum_{j=1}^{m} \lambda_j g_j(\boldsymbol{x}) - \sum_{j=m+1}^{n} \chi_j l_j(\boldsymbol{x}) \tag{29}$$

where λ_j and χ_j are the Lagrange multipliers. The λ_j can be positive or negative but the χ_j must be ≥ 0. In addition to Kuhn–Tucker conditions, the other necessary condition for $\boldsymbol{x}^*$ to be a local minimum of the problem, under consideration, is that the Hessian matrix of $\mathcal{L}$ should be positive semidefinite, that is,

$$\boldsymbol{v}^T \nabla^2 [(\boldsymbol{x}^*, \lambda^*, \chi^*)] \boldsymbol{v} \ge 0 \tag{30}$$

For a local minimum to be a global minimum, all the eigenvalues of the Hessian matrix should be ≥ 0.

A system of nonlinear equations is obtained, that can be solved using numerical methods such as a multivariable Newton–Raphson method. This method has been described in Stoecker [28] and applied by Culham and Muzychka [29], Culham et al. [30] and Khan et al. [31] to study the optimization of plate or pin fin heat sinks. In this study, the same approach is used to optimize the overall performance of a tube bank in such a manner that all relevant design conditions combine to produce the best possible tube bank for the given constraints. The optimized results are then compared for inline and staggered arrangements.

Table 1 Dimensions/data used for optimal design of tube banks

Quantity	Dimension/data
Cross-sectional area, mm^2	235×235
Length of tubes, mm	1000
Tube diameter, mm	12
Heat load, kW	20
Ambient temperature, K	300
Tube wall temperature, K	365

A simple procedure was coded in MAPLE 9, a symbolic mathematics software, which solves the system of N nonlinear equations using the multivariable Newton–Raphson method. Given $\mathcal{L}$, the solution vector $[\boldsymbol{x}]$, initial guess $[\boldsymbol{x_0}]$, and maximum number of iterations $N_{\max}$, the procedure systematically applies the Newton–Raphson method until the desired convergence criteria and/or maximum number of iterations is achieved. The method is quite robust provided an adequate initial guess is made.

IV. Results and Discussion

The objective of this study is to determine an optimal tube bank by minimizing the dimensionless entropy generation rate for different design variables including D, L, cross-sectional area of tube bank, $W \times H$, and Q. In each case, the optimum approach velocity/Reynolds number is determined corresponding to the minimum entropy generation rate. It is assumed that hot water is passed through the tubes, while air is passed in crossflow over the tubes. The ambient temperature and the tube wall temperatures are fixed at 300 and 365 K, respectively. The problem is solved for three different longitudinal and transverse pitch ratios and the overall performance is compared for both the inline and staggered arrangements.

The dimensions/data given in Table 1 are used as the default case to determine the performance parameters for both inline and staggered tube banks. In the first case, three different tube diameters 12, 13, and 14 mm are considered with the constraints of a maximum tube bank volume to dissipate a heat load of 20 kW. The problem is solved for each diameter corresponding to three dimensionless pitch ratios 1.25×1.25, 1.5×1.5, and 2.0×2.0. The results of optimization for the dimensionless heat transfer rate, pressure drop, and the number of tubes are tabulated in Tables 2 and 3 for each arrangement.

These results show that the number of tubes in a given volume decreases with the increase in tube diameter and/or dimensionless pitch ratio. In the inline arrangement, the heat transfer increases with the increase in tube diameter and/or dimensionless pitch ratio, whereas in the staggered arrangement, the heat transfer increases with the tube diameter but decreases with the increase in dimensionless pitch ratio. In both arrangements, the pressure drop increases with an increase in tube diameter but decreases with an increase in dimensionless pitch ratio.

Table 2 Results of optimization for inline tube banks

Dimensionless pitch ratio $\mathcal{S}_T \times \mathcal{S}_L$	Tube diameter, mm	Optimum approach velocity, m/s	Number of tubes $N_T \times N_L$	Nu_D	ΔP, Pa	$N_s \times 10^{10}$
1.25×1.25	12	3.4	15×15	88.4	590.2	0.180
	14	3.8	13×13	100.9	621.6	0.191
	16	4.2	11×11	113.1	650.3	0.201
1.5×1.5	12	5.7	13×13	88.5	480.4	0.251
	14	6.4	11×11	101.0	507.3	0.266
	16	7.0	10×10	112.9	532.8	0.281
2.0×2.0	12	9.6	10×10	91.5	452.7	0.365
	14	10.6	8×8	104.0	477.7	0.389
	16	11.6	7×7	116.0	499.9	0.410

Table 3 Results of optimization for staggered tube banks

Dimensionless pitch ratio $\mathcal{S}_T \times \mathcal{S}_L$	Tube diameter, mm	Optimum approach velocity, m/s	Number of tubes $N_T \times N_L$	Nu_D	ΔP, Pa	$N_s \times 10^{10}$
1.25×1.25	12	2.8	15×15	122.2	657.6	0.179
	14	3.2	13×13	142.0	660.8	0.180
	16	3.6	11×11	161.7	664.5	0.181
1.5×1.5	12	5.1	13×13	105.5	535.1	0.254
	14	5.8	11×11	121.8	544.1	0.259
	16	6.6	10×10	137.9	553.1	0.265
2.0×2.0	12	8.4	10×10	90.7	580.3	0.441
	14	9.4	8×8	104.0	597.6	0.458
	16	10.5	7×7	116.8	612.5	0.473

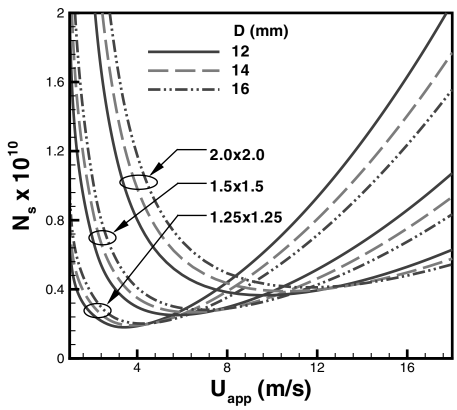

Fig. 2 Dimensionless entropy generation rate for inline arrangement.

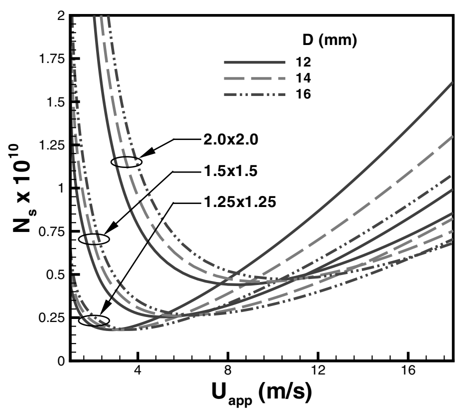

Fig. 3 Dimensionless entropy generation rate for staggered arrangement.

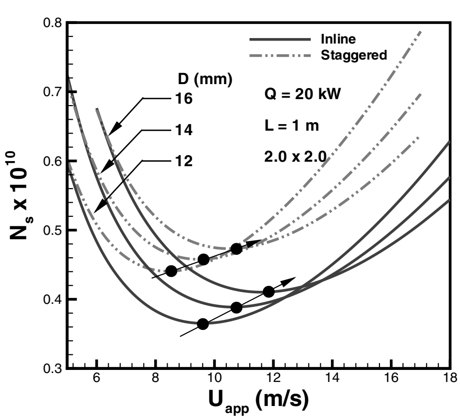

Fig. 4 Comparison of inline arrangement for a given tube diameter.

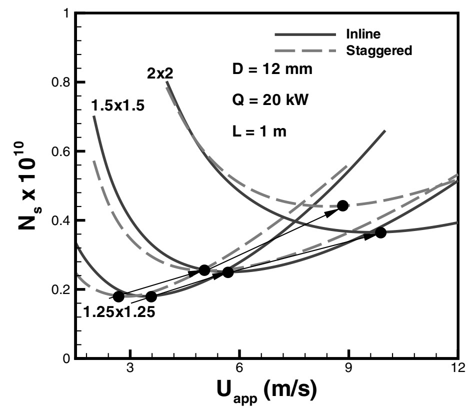

Fig. 5 Comparison of staggered arrangement for a given pitch ratio.

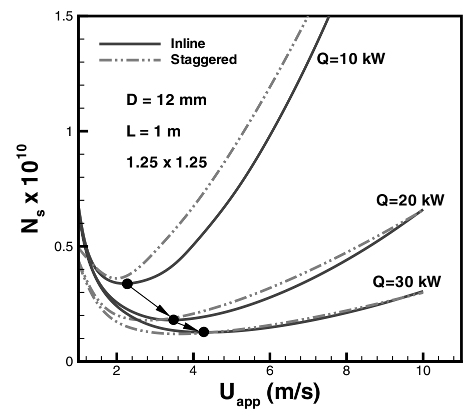

Fig. 6 Comparison of inline arrangement for 1.25 × 1.25.

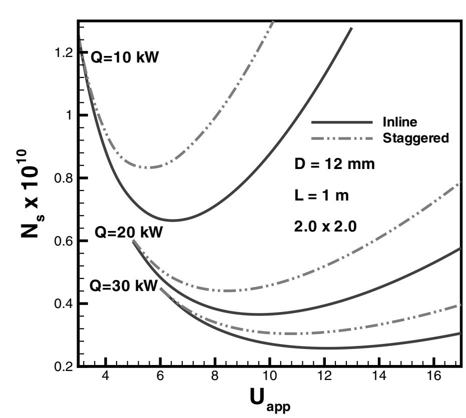

Fig. 7 Comparison of staggered arrangement for 2.0 × 2.0.

Results of optimization are also shown in Figs. 2 and 3 for each arrangement. These figures show that each arrangement has an optimum approach velocity for each tube diameter and for each dimensionless pitch ratio. As the tube diameter and/or the dimensionless pitch ratio increases, the optimum approach velocity as well as the dimensionless entropy generation rate increase.

For a given dimensionless pitch ratio 2.0×2.0, both arrangements are compared in Fig. 4 for three tube diameters. It shows that the optimum approach velocity increases with the tube diameter for both arrangements. The dimensionless entropy generation rate is higher for the staggered arrangement. In Fig. 5, both arrangements are compared for a given tube diameter. It shows that the staggered arrangement gives better performance for lower pitch ratios and lower approach velocities but for higher approach velocities and widely spaced tube banks, the inline arrangement is better with reference to lowest dimensionless entropy generation rate. Optimum approach velocities are also found to be lower for a staggered arrangement in the case of compact banks. Figures 6 and 7 show the effects of heat load on the performance of compact and widely spaced tube banks for both arrangements. The optimum dimensionless entropy generation rate decreases with the increase in heat load and the dimensionless pitch ratio for both arrangements. In the case of compact tube banks, the staggered arrangement gives better performance for lower approach velocities only, whereas the inline arrangement is better for higher approach velocities and also for widely spaced banks. The optimum approach velocities increase with heat loads for both type of banks.

The effects of tube length on the performance of tube banks are shown in Figs. 8 and 9 for both arrangements. Figure 8 shows those effects for a compact tube bank. It shows that the effect of tube length is almost negligible on the optimum dimensionless entropy

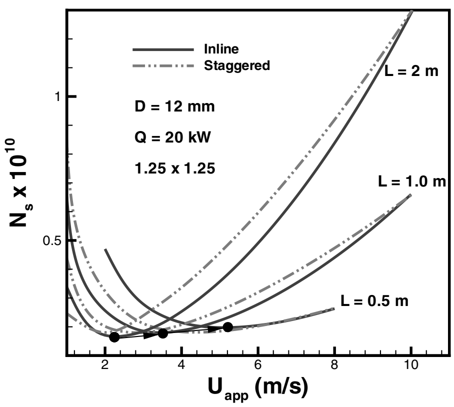

Fig. 8 Effect of tube length on dimensionless entropy generation rate for 1.25 × 1.25.

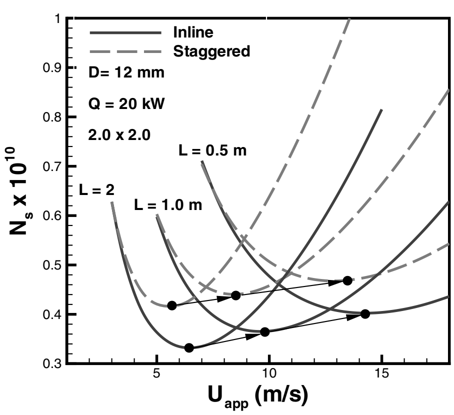

Fig. 9 Effect of tube length on dimensionless entropy generation rate for 2.0 × 2.0.

generation rate but the optimum approach velocity decreases with the increase in tube length for both arrangements. In the case of widely spaced tube bank (Fig. 9), the inline arrangement performs much better for all three cases. Again, the optimum approach velocity decreases with an increase in tube length for both arrangements. Figure 10 shows the effects of Reynolds number on the performance of tube banks for both arrangements. For different dimensionless bank heights, inline arrangements give a better performance for lower Reynolds numbers, but as the Reynolds number increases, the staggered arrangements do a good job for the widely spaced tube bank. It shows that the optimum Reynolds number increases with a decrease in the cross-sectional area.

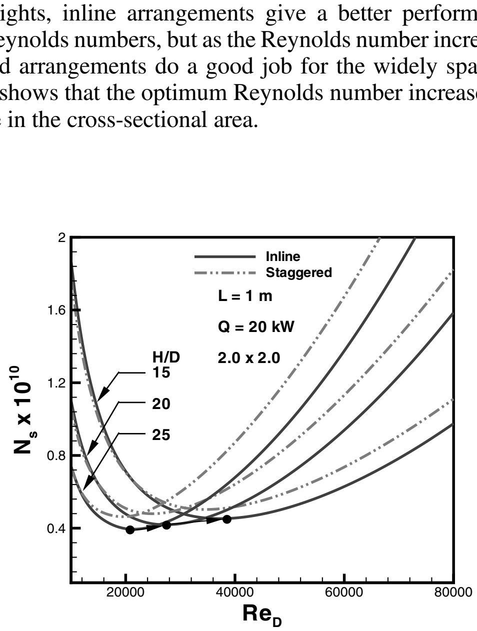

Fig. 10 Effect of the Reynolds number on dimensionless entropy generation rate for a widely spaced tube bank.

V. Conclusions

A scientific procedure is presented for determining an optimal design of tube banks for both inline and staggered arrangements. The effects of tube diameter, tube length, dimensionless pitch ratios, front cross-sectional area of the tube bank, and heat load are examined with respect to their role in influencing optimum design conditions and the overall performance of the tube bank. It is demonstrated that the staggered arrangement gives a better performance for lower approach velocities and longer tubes, whereas the inline arrangement performs better for higher approach velocities and larger dimensionless pitch ratios. Compact tube banks perform better for both arrangements and for smaller tube diameters.

Acknowledgments

The authors gratefully acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada and the Centre for Microelectronics Assembly and Packaging.

References

[1] McClintock, F. A., "The Design of Heat Exchangers for Minimum Irreversibility," ASME Paper 51-A-108, 1951.

[2] Bejan, A., "The Concept of Irreversibility in Heat Exchanger Design: Counterflow Heat Exchangers for Gas-to- Gas Applications," *Journal of Heat Transfer*, Vol. 99, Aug. 1977, pp. 375–380.

[3] Bejan, A., "General Criteria for Rating Heat Exchanger Performance," *International Journal of Heat and Mass Transfer*, Vol. 21, No. 5, 1978, pp. 655–658.

[4] Bejan, A., "A Study of Entropy Generation in Fundamental Convective Heat Transfer," *Journal of Heat Transfer*, Vol. 101, Nov. 1979, pp. 718–725.

[5] Bejan, A., *Entropy Generation Through Heat and Fluid Flow*, John Wiley & Sons, New York, 1982.

[6] Aceves-Saborio, S., Ranasinghe, J., and Reistad, G. M., "An Extension to the Irreversibility Minimization Analysis Applied to Heat Exchangers," *Journal of Heat Transfer*, Vol. 111, No. 1, 1989, pp. 29–36.

[7] Grazzini, G., and Gori, F., "Entropy Parameters for Heat Exchanger Design," *International Journal of Heat and Mass Transfer*, Vol. 31, No. 12, 1988, pp. 2547–2554.

[8] Sekulic, D. P., "The Second Law Quality of Energy Transformation in a Heat Exchanger," *Journal of Heat Transfer*, Vol. 112, May 1990, pp. 295–300.

[9] Zhang, L. W., Balachandar, S., Tafti, D. K., and Najjar, F. M., "Heat Transfer Enhancement Mechanisms in In-line and Staggered Parallel-Plate Fin Heat Exchangers," *International Journal of Heat and Mass Transfer*, Vol. 40, No. 10, 1997, pp. 2307–2325.

[10] Ordonez, J. C., and Bejan, A., "Entropy Generation Minimization in Parallel-Plates Counterflow Heat Exchangers," *International Journal of Energy Research*, Vol. 24, No. 10, 2000, pp. 843–864.

[11] Bejan, A., "Thermodynamic Optimization of Geometry in Engineering Flow Systems," *Energy*, Vol. 1, No. 4, 2001, pp. 269–277.

[12] Bejan, A., "Fundamentals of Energy Analysis, Entropy Generation Minimization, and the Generation of Flow Architecture," *International Journal of Energy Research*, Vol. 26, No. 7, 2002, pp. 545–565.

[13] Yilmaz, M., Sara, O. N., and Karsli, S., "Performance Evaluation Criteria for Heat Exchangers Based on Second Law Analysis," *Energy*, Vol. 1, No. 4, pp. 278–294.

[14] Unuvarn, A., and Kargici, S., "An Approach for the Optimum Design of Heat Exchangers, " *International Journal of Energy Research*, Vol. 28, No. 15, pp. 1379–1392.

[15] Peters, M., and Timmerhaus, K., *Plant Design and Economics for Chemical Engineers*, 4th ed., McGraw–Hill, Singapore, 1991.

[16] Shah, R. K., Afimiwala, K. A., and Mayne, R. W., "Heat Exchanger Optimization," *Proceedings of the Sixth International Heat Transfer Conference*, Vol. 4, Hemisphere Publishing Corporation, Washington, DC, 1978, pp. 185–191.

[17] Shah, R. K., "Compact Heat Exchanger Surface Selection Methods," *Proceedings of Sixth International Heat Transfer Conference*, Vol. 4, Hemisphere Publishing Corporation, Washington, DC, 1978, pp. 193–199.

[18] Van Den Bulck, E., "Optimal Design of Crossflow Heat Exchangers," *Journal of Heat Transfer*, Vol. 113, No. 2, 1991, pp. 341–347.

[19] Poulikakos, D., and Bejan, A., "Fin Geometry for Minimum Entropy Generation in Forced Convection," *Journal of Heat Transfer*, Vol. 104, Nov. 1982, pp. 616–623.

[20] Khan, W. A., Culham, J. R., and Yovanovich, M. M., "The Role of Fin Geometry in Heat Sink Performance," presented at *International Electronic Packaging Technical Conference and Exhibition*, American Society of Mechanical Engineers, New York, 2003; also *Journal of Electronic Packaging* (to be published).

[21] Bejan, A., "Entropy Generation Minimization: The New Thermodynamics of Finite-Size Devices and Finite-Time Processes," *Journal of Applied Physics*, Vol. 79, No. 3, 1996, pp. 1191–1218.

[22] Stanescu, G., Fowler, A. J., and Bejan, A., "The Optimal Spacing of Cylinders in Free-Stream Crossflow Forced Convection," *International Journal of Heat and Mass Transfer*, Vol. 39, No. 2, 1996, pp. 311–317.

[23] Matos, R. S., Vargas, J. V. C., Laursen, T. A., and Saboya, F. E. M., "Optimization Study and Heat Transfer Comparison of Staggered Circular and Elliptic Tubes in Forced Convection," *International Journal of Heat and Mass Transfer*, Vol. 44, 2001, pp. 3953–3961.

[24] Vargas, Jose, V. C., Bejan, A., and Siems, D. L., "Integrative Thermodynamic Optimization of the Crossflow Heat Exchanger for an Aircraft Environmental Control System," *Journal of Heat Transfer*, Vol. 123, Aug. 2001, pp. 760–769.

[25] Khan, W. A., Culham, J. R., and Yovanovich, M. M., "Convection Heat Transfer from Tube Banks in Crossflow: Analytical Approach," presented at *Proceedings of the 43rd AIAA Aerospace Sciences Meeting and Exhibit*, American Institute of Aeronautics and Astronautics, Inc., Reston, VA, 2005; also *Journal of Thermophysics and Heat Transfer* (in press).

[26] Žukauskas, A., and Ulinskas, R., "Single-Phase Fluid Flow: Banks of Plain and Finned Tubes," *HEDH, Heat Exchanger Design Handbook*, Washington Hemisphere Publishing, New York, 1983, Chap. 2.2.4.

[27] Khan, W. A., "Modeling of Fluid Flow and Heat Transfer for Optimization of Pin–Fin Heat Sinks," Ph. D. Thesis, Department of Mechanical Engineering, University of Waterloo, Canada, 2004.

[28] Stoecker, W. F., *Design of Thermal Systems*, McGraw–Hill, New York, 1989.

[29] Culham, R. J., and Muzychka, Y. S., "Optimization of Plate Fin Heat Sinks Using Entropy Generation Minimization," *IEEE Transactions on Components and Packaging Technologies*, Vol. 24, No. 2, pp. 159–165, 2001.

[30] Culham, R. J., Khan, W. A., Yovanovich, M. M., and Muzychka, Y. S., "The Influence of Material Properties and Spreading Resistance in the Thermal Design of Plate Fin Heat Sinks," *Proceedings of the 35th National Heat Transfer Conference*, American Society of Mechanical Engineers, New York, 2001, pp. 240–246.

[31] Khan, W. A., Culham, J. R., and Yovanovich, M. M., "Optimization of Pin-Fin Heat Sinks Using Entropy Generation Minimization," *IEEE Transactions on Components and Packaging Technologies*, Vol. 28, No. 2, pp. 247–254, 2005; also *Proceedings of ITherm 2004*, 2004.